A Generalized Methodology for Estimating Minimum Fluidization Velocity at Elevated Pressure and Temperature

A generalized methodology for estimating minimum fluidization velocity at elevated pressure and temperature was developed on the basis of a general correlation for pressure drop through fixed beds of spherical particles proposed by Barnea and Mizrahi (1973) and extended by Barnea and Mednick (1975). Comparison with experimental results shows good agreement.

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SCOPE

The minimum fluidization velocity is a fundamental characteristic of a fluidized bed. Its accurate prediction is important for successful design and operation of a fluidized bed. There are numerous studies and proposed correlations on the prediction of the minimum fluidization velocity at ambient conditions (Babu et al., 1978; Grewal and Saxena, 1980). Their extrapolation to elevated pressure and temperature is uncertain, however. The general approach is to estimate the effect of pressure and temperature by employing Ergun's (1952) equation or its variations, such as that suggested by Wen and Yu (1966). Since the correct shape factor of the particles and the voidage to be used in the Ergun equation are generally not available, the effect of shape factor and voidage are thus lumped into two separate constants. So far no less than five sets of constants have been proposed in the literature with various degrees of success. The accuracy is uncertain for particles with characteristics outside the range of the data correlated by different authors. Undoubtedly, more sets of constants will be proposed later in the literature to take care of this difficulty. A more fundamental approach, extending the general correlation for pressure drop through fixed beds of spherical particles developed by Barnea and Mizrahi (1973), is described here.

CONCLUSIONS AND SIGNIFICANCE

A generalized methodology, extending the general correlation for pressure drop through fixed beds of spherical particles by Barnea and Mizrahi (1973), has been developed for prediction of minimum fluidization velocity at elevated pressure and temperature. The methodology successfully correlated the experimental minimum fluidization velocities obtained at the Pittsburgh Energy Technology Center (PETC) and in the literature using coal, char, ballotini, catalyst, and sand of sizes ranging from 88 to 3,376 μm as the bed material and at pressures up to 6,300 kPa and temperatures up to 1,123 K. The proposed methodology can also be applied to bed materials of relatively wide size distribution as long as the harmonic mean particle size is used. To apply the methodology, the minimum fluidization velocity of the system under study needs to be determined experimentally at ambient conditions. This usually is not a severe limitation for most applications.

The approach is believed to be fundamental and is generally applicable in all systems except those systems with particles substantially deviate from the spherical shape such as the graphite particles with a sphericity of 0.65. The methodology also cannot be applied to Geldart's class A powders (Geldart, 1973) where the voidage at minimum fluidization changes substantially with pressure. The methodology allows accurate prediction of the minimum fluidization velocity at elevated pressure and temperature hitherto unavailable.

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INTRODUCTION

The effect of pressure and temperature on the U_{mf} was usually estimated using Ergun's equation or its variations, such as that suggested by Wen and Yu (1966). The Ergun equation can be written as

$$\frac{\Delta P}{L} = \frac{150(1-\epsilon)^2 \mu U}{\phi^2 \epsilon^3 d_p^2} + \frac{1.75(1-\epsilon)U^2 \rho_f}{\epsilon^3 \phi d_p} \tag{1}$$

At the minimum fluidization condition, the pressure drop across the bed can be expressed as

$$\frac{\Delta P}{L} = (\rho_s - \rho_f)(1 - \epsilon_{mf})g \tag{2}$$

Equating Eqs. 1 and 2, a quadratic equation in $(Re)_{mf}$ can be obtained.

$$\underset{\text{(kinetic term)}}{1.75A(Re)_{mf}^2} + \underset{\text{(viscous term)}}{150B(Re)_{mf}} = \mathrm{Ar} \tag{3}$$

where

$$A = \frac{1}{\phi \epsilon_{mf}^3}; \; B = \frac{1-\epsilon_{mf}}{\phi^2 \epsilon_{mf}^3} \tag{4}$$

The shape-voidage functions A and B have been approximated by Wen and Yu (1966) to be

$$A \cong 14; B \cong 11 \tag{5}$$

Equation 3 can be rearranged into

$$(Re)_{mf} = \sqrt{(C_1)^2 + C_2 Ar} - C_1 \tag{6}$$

where

$$(Re)_{mf} = \frac{d_p U_{mf} \rho_f}{\mu}, \text{ the Reynolds number} \tag{7}$$

$$Ar = \frac{d_p^3 \rho_f (\rho_s - \rho_f) g}{\mu^2}, \text{ the Archimedes number} \tag{8}$$

The two constants, C_1 and C_2, in the original Wen and Yu equations are 33.7 and 0.0408, respectively. Several other sets of constants are also proposed. They are summarized below.

Reference	C_1	C_2
Wen and Yu (1966)	33.7	0.0408
Richardson (1971)	25.7	0.0365
Babu et al. (1978)	25.25	0.0651
Grace (1982)	27.2	0.0408
Chitester et al. (1984)	28.7	0.0494

For low Reynolds number where $(Re)_{mf} < 20$, U_{mf} can be calculated from Eq. 3 by neglecting the kinetic term.

$$U_{mf} = \frac{d_p^2(\rho_s - \rho_f)g}{1{,}650\mu} \text{ for } (\mathrm{Re})_{\mathrm{mf}} < 20 \tag{9}$$

Since the gas viscosity is usually independent of the pressure and the particle density is much larger than the fluid density, $\rho_s \gg \rho_f$, in most cases, the U_{mf} of small particles do not usually change with the pressure. For large Reynolds number, the viscous term is negligible and

$$U_{mf}^2 = \frac{d_p(\rho_s - \rho_f)g}{24.5\rho_f} \text{ for } (Re)_{mf} > 1{,}000 \tag{10}$$

For large or dense particles, increasing the pressure will reduce the minimum fluidization velocity of the particles.

If the voidage at minimum fluidization and the particle shape are known, the original Ergun equation (1952) gives a better prediction than its simplified variations. Unfortunately, in most instances, neither information is available. The accuracy of U_{mf} predictions based on the Ergun type of equations depends on the selection of the constants C_1 and C_2 in Eq. 6. Several sets of constants C_1 and C_2 have been suggested by different authors based on different data bases as summarized earlier. In view of so many sets of constants already suggested so far, it is safe to expect that more sets of constants will be proposed in the future for different data bases. For fundamental predictions of the fundamental fluidization property such as the minimum fluidization velocity, a more fundamental approach is necessary. Other correlations available in the literature for prediction of the minimum fluidization velocity have been surveyed by Babu et al. (1978) and Grewal and Saxena (1980).

EXPERIMENTAL APPARATUS

Experiments were conducted utilizing a high-pressure fluidization cold model facility. The facility comprises a pressure vessel 61 cm in diameter, a nitrogen supply system, and peripheral equipment, as shown in Figure 1. The pressure vessel, capable of operating at pressures up to 6,895 kPa, is 3.66 m high and is equipped with 24 glass observation ports. Each observation port has a viewing diameter of 8.57 cm. Plexiglas fluidization vessels of two-dimensional and three-dimensional geometries can be inserted into the pressure vessel for study. The interior of the vessel is lighted by internal quartz-halogen lamps and external spotlights. Hinged closures are used as end caps on the pressure vessel to allow quick access to the internals. The nitrogen supply system uses 24 high-pressure gas accumulators. Each 1.39 m^3 accumulator is pressure-rated at 16,200 kPa. No nitrogen recirculation system is used. Supply pressure in the accumulators is maintained by a central liquid nitrogen system. Flow control is accomplished by use of a parallel control valve-rotameter setup. The control valves (FCV 1-1, 1-2 in Figure 1) are used for large flows, and the rotameters are used for small flows. A gas meter mounted inside the pressure vessel allows for frequent calibration of the flow-measuring instrumentation. An upstream pressure control valve (PCV-1) and a back pressure control valve (PCV-2) are used to maintain the system pressure.

Experimental Conditions

The minimum fluidization velocities for coal, char, and ballotini with different particle size distributions were determined in a circular Plexiglas model 10.16 cm in diameter inserted into the pressure vessel described earlier. A sintered stainless steel porous plate was used as a distributor. The bed expansion and the voidage at minimum fluidization were also recorded. The characteristics of the bed material employed during the experiments are summarized in Table 1. It is important to note that the size distributions of some of the bed materials are quite wide. The experiments were performed at four different pressure levels for each size distribution: atmospheric pressure, and 2,169, 4,238, and 6,306 kPa.

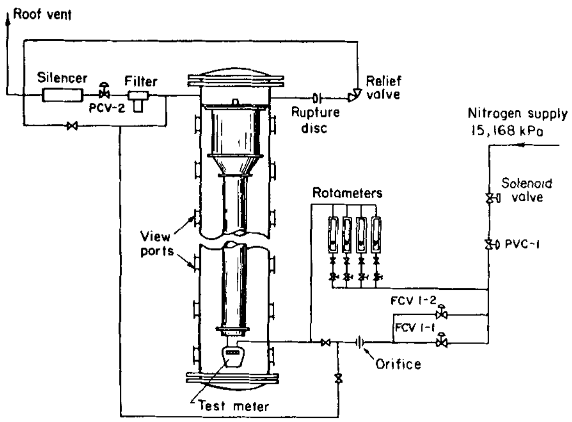

Figure 1. Schematic diagram of high-pressure fluidization cold model facility.

TABLE 1. PARTICLE SIZE DISTRIBUTION OF BED MATERIALS EMPLOYED IN EXPERIMENTS

Bed Material	Particle Density kg/m³	Particle Size Distribution, Weight Fraction (U.S. Mesh Size) −9 + 14	−14 + 28	−28 + 35	−35 + 48	−48 + 60	−60 + 80	−80 + 100	−100 + 150	−150 + 200	−200 + 270	−270 + 325	−325 + 400	Harmonic Mean Dia.,* μm
Coal	1,247	—	—	—	0.010	0.012	0.017	0.014	0.296	0.430	0.140	0.056	0.025	88
Coal	1,247	—	0.005	0.095	0.163	0.073	0.137	0.102	0.139	0.171	0.090	0.017	0.008	140
Coal	1,247	—	—	—	0.015	0.087	0.425	0.281	0.188	0.004	—	—	—	179
Coal	1,247	—	0.017	0.219	0.566	0.163	0.034	0.001	—	—	—	—	—	358
Coal	1,247	—	0.013	0.268	0.520	0.164	0.030	0.003	0.002	—	—	—	—	361
Char	1,116	0.002	0.025	0.052	0.140	0.111	0.195	0.143	0.082	0.204	0.046	—	—	157
Char	1,116	0.007	0.045	0.069	0.162	0.123	0.233	0.144	0.125	0.088	0.002	0.001	0.001	196
Char	1,116	0.015	0.170	0.218	0.368	0.167	0.046	0.003	0.003	0.003	0.003	0.002	0.002	374
Ballotini	2,472	—	—	—	0.001	0.017	0.420	0.165	0.062	0.018	0.138	0.112	0.067	96
Ballotini	2,472	—	—	—	0.002	0.001	0.006	0.022	0.530	0.346	0.071	0.019	0.003	102

* Harmonic Mean Particle Diameter = $1/\Sigma_i X_i/d_{pi}$

DEVELOPMENT OF A CORRELATION FOR ESTIMATING MINIMUM FLUIDIZATION VELOCITY

It was pointed out earlier that use of the Wen and Yu equation (1966) for calculating the minimum fluidization velocity at elevated pressure and temperature is not satisfactory in that more than five sets of empirical constants have been proposed for use in the equation. The accuracy is uncertain for particles with characteristics outside the range of the data correlated by different authors. *An approach with more consistent results is required.*

Although the Wen and Yu equation has been widely used in the fluidization literature, it is hardly recognized that the equation is simply an expression of Reynolds number—drag coefficient (Re–C_D) relationship, first pointed out by Zenz and Othmer (1960). By equating the drag force and the gravitational force for a single spherical particle, we have

$$\frac{\pi}{8} C_D \rho_f U^2 d_p^2 = \frac{\pi d_p^3}{6} (\rho_s - \rho_f) g \tag{11}$$

or

$$C_D = \frac{4}{3} \frac{d_p(\rho_s - \rho_f) g}{\rho_f U^2} \tag{12}$$

It can be readily found that

$$Ar = \frac{4}{3} \mathrm{Re}^2 \mathrm{C_D} \text{ for spherical particles} \tag{13}$$

The Wen and Yu type of equation shown in Eq. 6 can be rearranged to give

$$C_D = \frac{(Re + C_1)^2 - C_1^2}{\frac{3}{4} C_2 Re^2} \tag{14}$$

Equation 14 is simply another expression for the relationship between Re and C_D.

In 1973 Barnea and Mizrahi proposed a general correlation for pressure drop through fixed beds of spherical particles based on a discrete particle model (rather than a pipe flow analogy) corrected for particle interaction. They extended the standard C_D vs. Re curve for single sphere to multiparticle systems by incorporating proper functions of the volumetric particle concentration. The modified Reynolds number and drag coefficient they suggested are

$$(Re)_\epsilon = Re \left[\frac{1}{\epsilon \exp\{5(1-\epsilon)/3\epsilon\}}\right] \tag{15}$$

$$(C_D)_\epsilon = C_D \frac{\epsilon^3}{1 + (1-\epsilon)^{1/3}} \tag{16}$$

By substituting the standard Re–C_D curve for a single sphere with the modified $(Re)_\epsilon$ and $(C_D)_\epsilon$ expressed in Eqs. 15 and 16, the standard Re–C_D can be applied for multispherical particle systems. The suggested correlation was extended by Barnea and Mednick (1975) for calculation of the minimum fluidization velocity. For

TABLE 2. RECOMMENDED DRAG CORRELATIONS: STANDARD DRAG CURVE, $w = \log_{10} Re$*

Range	Correlation
(A) $Re < 0.01$	$C_D = 3/16 + 24/Re$
(B) $0.01 < Re \leq 20$	$\log_{10}\left[\frac{C_D Re}{24} - 1\right] = -0.881 + 0.82w - 0.05w^2$
	i.e., $C_D = \frac{24}{Re}[1 + 0.1315\, Re^{(0.82-0.05w)}]$
(C) $20 < Re \leq 260$	$\log_{10}\left[\frac{C_D Re}{24} - 1\right] = -0.7133 + 0.6305w$
	i.e., $C_D = \frac{24}{Re}[1 + 0.1935\, Re^{0.6305}]$
(D) $260 \leq Re \leq 1{,}500$	$\log_{10} C_D = 1.6435 - 1.1242w + 0.1558w^2$
(E) $1.5 \times 10^3 \leq Re \leq 1.2 \times 10^4$	$\log_{10} C_D = -2.4571 + 2.5558w - 0.9295w^2 + 0.1049w^3$
(F) $1.2 \times 10^4 < Re < 4.4 \times 10^4$	$\log_{10} C_D = -1.9181 + 0.6370w - 0.0636w^2$

* From Clift et al. (1978), p. 112.

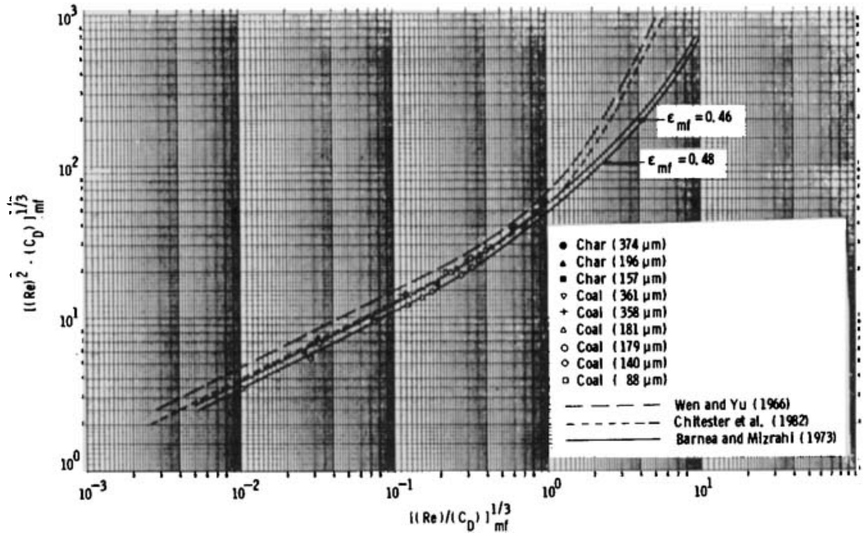

Figure 2. Comparison of experimental data with Re–C_D curve for coal and char particles.

TABLE 3. PRESSURE EFFECT ON BED VOIDAGE AT MINIMUM FLUIDIZATION FOR VARIOUS PARTICLES

Substance	Harmonic Mean Dia., μm	ϵ_{mf} 101 kPa	2,169 kPa	4,238 kPa	6,306 kPa
Coal	88	0.49	0.50	0.51	0.50
	140	0.45	0.45	0.46	0.46
	179	0.46	0.46	0.47	*
	182	*	0.45	0.45	*
	358	0.44	0.46	0.46	0.47
	361	0.44	0.47	0.47	0.47
Char	157	0.51	0.53	0.53	0.53
	196	0.50	*	0.53	0.53
	374	0.55	0.57	0.58	0.58
Ballotini	96	0.40	0.41	0.42	0.42
	102	0.45	0.45	0.44	0.45

** Data not available.

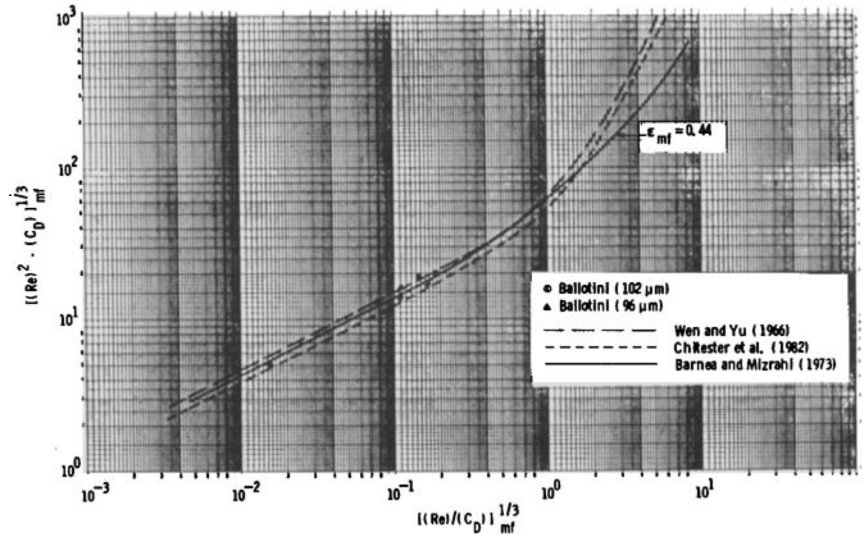

Figure 3. Comparison of experimental data with Re–C_D curve for ballotini particles.

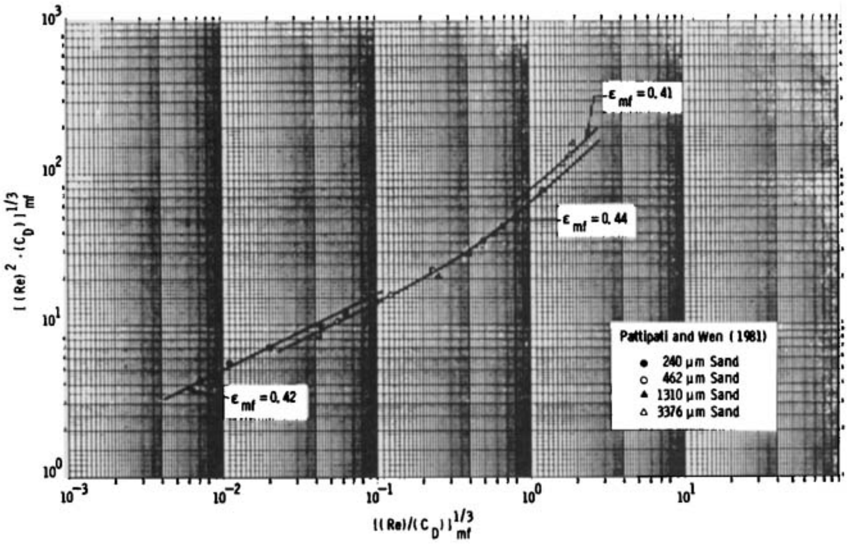

Figure 4. Comparison of correlation with minimum fluidization velocity data at elevated temperature by Pattipati and Wen (1981).

easy graphical presentation, they suggested plotting the dimensionless diameter $(Re^2 \cdot C_D)_{mf}^{1/3}$ vs. the dimensionless velocity $(Re/C_D)_{mf}^{1/3}$. At minimum fluidization condition, these two dimensionless groups can be expressed as

$$\left(\frac{(Re)_\epsilon}{(C_D)_\epsilon}\right)_{mf}^{1/3} = \left(\frac{Re}{C_D}\right)_{mf}^{1/3} \left\{ \frac{[1 + (1 - \epsilon_{mf}^{1/3})]^{1/3}}{\epsilon_{mf}^{4/3} \exp\left(\dfrac{5}{9}\dfrac{(1 - \epsilon_{mf})}{\epsilon_{mf}}\right)} \right\} \quad (17)$$

$$[(Re)_\epsilon^2 (C_D)_\epsilon]_{mf}^{1/3} = (Re^2 C_D)_{mf}^{1/3} \times \left\{ \frac{\epsilon_{mf}^{1/3}}{[1 + (1 - \epsilon_{mf})^{1/3}]^{1/3} \exp\left(\dfrac{10(1 - \epsilon_{mf})}{9\epsilon_{mf}}\right)} \right\} \quad (18)$$

In this study, Eqs. 17 and 18 were combined with the standard drag correlations recommended by Clift et al. (1978), which are summarized in Table 2, for comparison with the experimental minimum fluidization velocities obtained at elevated pressure and temperature.

COMPARISON WITH EXPERIMENTAL DATA AT ELEVATED PRESSURE

The experimental minimum fluidization velocity data obtained at the Pittsburgh Energy Technology Center (PETC) using different sizes of coal, char, and ballotini as bed material and operated at pressures of 101, 2,169, 4,238 and 6,306 kPa are compared with that calculated from Eqs. 17 and 18 at a selected voidage in Figures 2 and 3. The Wen and Yu correlation (1966) and the correlation by Chitester et al. (1983) are also included for comparison. The coal sizes of harmonic mean diameters of 88 μm and 140 μm can best be correlated with the curve for $\epsilon_{mf} = 0.48$; see Figure 2. The experimentally observed voidages at minimum fluidization for these particles range from 0.45 to 0.51 as shown in Table 3. The ballotini particles can be correlated fairly well with the curve for $\epsilon_{mf} = 0.44$, Figure 3. This compares with the experimental voidages of 0.40 to 0.45. The rest of the data can be correlated with the curve for $\epsilon_{mf} = 0.46$ as compared to experimental values of 0.44 to 0.47 (except that of char particles). All char particles have consistently higher experimental voidage at minimum fluidization. It is suspected that the difficulty in determining the particle density of the

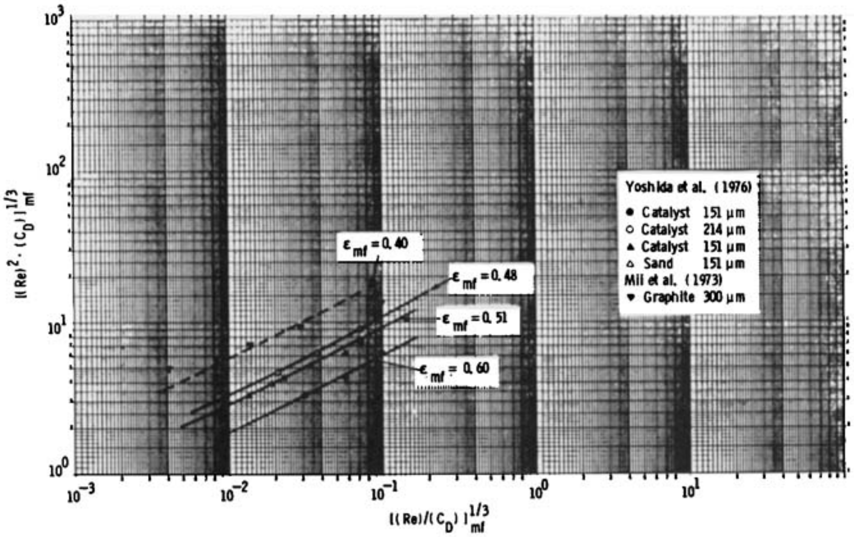

Figure 5. Comparison of correlation with minimum fluidization velocity data at elevated temperature by Yoshida et al. (1976).

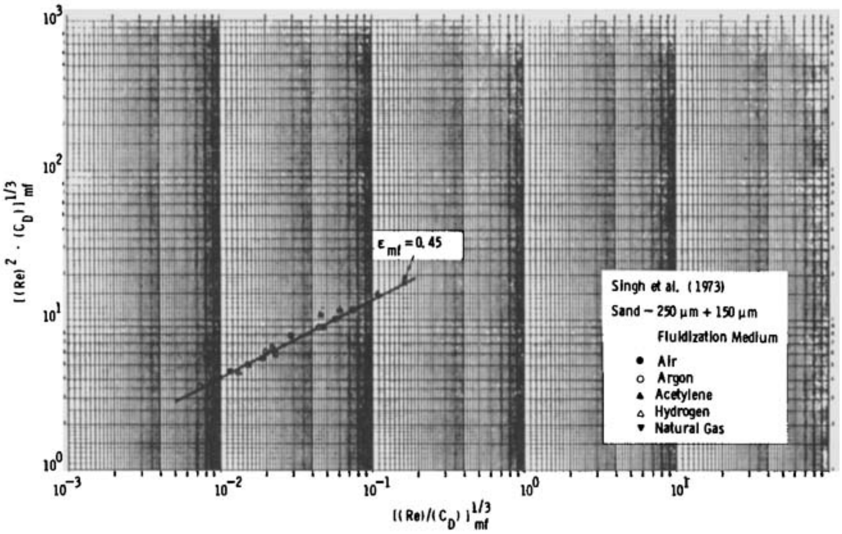

Figure 6. Comparison of correlation with minimum fluidization velocity data at elevated temperature by Singh et al. (1973).

porous char particles has contributed to this consistent discrepancy. The curve for $\epsilon_{mf} = 0.53$ will account for char particles of density of 0.55 g/cm^3, a value consistent with char densities reported in the literature.

The fact that the minimum fluidization velocities at different pressures for the same particles can be correlated with a single curve at a constant ϵ_{mf} provides a simple methodology for estimating the minimum fluidization velocity at elevated pressure accurately. The following procedure is suggested. The material of interest is first subjected to experimentation at atmospheric pressure to determine its minimum fluidization velocity. This data point is then located on the proper curve of constant ϵ_{mf} in a plot similar to that shown in Figures 2 and 3. The minimum fluidization velocities at any pressure for the same material can then be calculated readily from the same curve.

The coal, char, and ballotini particles employed by PETC for minimum fluidization velocity studies are by no means of narrow size distribution. The harmonic mean particle sizes are used for the present analysis. The harmonic mean particles sizes evaluated as shown in Table 1 are equivalent to the surface-to-volume mean particle sizes. There is legitimate justification for using the harmonic mean particle size in the discrete particle model employed here because the drag on the particles is proportional to the specific particle surface area per unit bed volume. The harmonic mean particle size can be theoretically derived to be the reciprocal of the specific particle surface area per unit bed volume. The successful correlation obtained with materials of reasonably wide size distribution here implies that the proposed methodology can also be applied for bed materials of relatively wide size distribution as long as the harmonic mean particle size is used in the correlation.

The curves shown in Figures 2 and 3 are obtained from the Re–C_D relationship for spherical particles. For particles of irregular shape, the correction for Re–C_D is available. The determination of shape factor for different materials is difficult. This is especially true when particles of wide size distribution are involved where the shape of the particles may be different for different sizes of particles. Judging from the fact that the experimental voidages at minimum fluidization conditions seem to agree fairly well with the voidages of the particular curves fitting the particular sets of data as shown in Figures 2 and 3, the effect of the particle shape may very well be of second order when the bed is at its most loosely packed condition at minimum fluidization, except for particles substantially deviate from the spherical particles.

COMPARISON WITH EXPERIMENTAL DATA AT ELEVATED TEMPERATURE

The fundamental correlation originally developed for estimating the U_{mf} at elevated pressure was also found to be applicable at elevated temperature. The correlation was applied to four sets of minimum fluidization velocity data obtained at elevated temperature, those by Singh et al. (1973), Mii et al. (1973), Yoshida et al. (1976), and Pattipati and Wen (1981). They are presented in Figures 4 through 6 with very encouraging results. The experimental conditions and the particle characteristics employed by these authors are summarized in Table 4. It can now be concluded that the correlation and the methodology proposed to estimate the

TABLE 4. SUMMARY OF MINIMUM FLUIDIZATION VELOCITY DATA AT ELEVATED TEMPERATURE EMPLOYED FOR TESTING THE CORRELATION

References	Bed Material: Material	Size μm	Density kg/m^3	Fluidizing Medium	Temp. K
Mii et al. (1973)	Graphite	300	1,970	Air and nitrogen	293–1,073
Singh et al. (1973)	Sand	−250 + 150	2,650	Air, argon, steam, acetylene, hydrogen, natural gas	288–973
Yoshida et al. (1976)	Catalyst	151, 214	1,540	Air	293–673
	Sand	151	2,580	Air	293–673
Pattipati and Wen (1981)	Sand	240	2,630	Air	293–1,073
	Sand	462	2,630	Air	293–1,023
	Sand	1,310	2,630	Air	373–1,123
	Sand	3,376	2,630	Air	318–773

minimum fluidization velocity at elevated pressure is a fundamental approach and can be equally applicable for estimating the minimum fluidization velocity at elevated temperature for spherical particles and relatively roundish particles such as catalyst and sand. For particles substantially deviate from the spherical particles such as the graphite particles with a sphericity of 0.65 used by Mii et al. (1973), the correlation does not apply as well without further correction with nonsphericity.

PROPOSED GENERALIZED METHODOLOGY

Combining Eqs. 17 and 18 and the recommended drag relationships proposed by Clift et al. (1978), a series of curves can be obtained by plotting the dimensionless diameter $[(Re)^2(C_D)]_{mf}^{1/3}$ vs. the dimensionless velocity $[(Re)/(C_D)]_{mf}^{1/3}$ with the voidage as a parameter as shown in Figures 2 to 6. The minimum fluidization velocity of the bed material of interest is first determined at ambient conditions, i.e., atmospheric pressure and room temperature. This data point is then located on the proper curve of constant ϵ_{mf} in a plot similar to Figures 2 to 6. Once this applicable curve of constant voidage is identified, the minimum fluidization velocities at any pressure or temperature can be calculated readily from the same curve.

The success of this approach implies that the voidage at minimum fluidization does not change appreciably with respect to the pressure and the temperature. There are conflicting reports in the literature on the constancy of the voidage at minimum fluidization when the pressure and the temperature are varied (Botterill et al., 1982; Pattipati and Wen, 1982; Rowe et al., 1982; King, 1979). The variation of the voidage at minimum fluidization with respect to changes in pressure and temperature reported in the literature is very small, however, if at all for Geldart's class B and D powders. The proposed methodology represents the most general and accurate method for predicting the minimum fluidization velocity at elevated pressure and temperature among the correlations reported in the literature so far. The necessity of determining the minimum fluidization velocity at ambient conditions does not constitute a severe limitation for most applications. For Geldart's class A powder where the voidage at minimum fluidization changes appreciably with pressure and for particles substantially deviate from the spherical particles, the methodology does not apply as well without further corrections.

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Reference in this report to any specific commercial product, process, or service is to facilitate understanding and does not necessarily imply its endorsement or favoring by the U.S. Department of Energy.

NOTATION

A,B = constants in Eqs. 3 and 4
Ar = Archimedes number, $Ar = d_p^3\rho_f(\rho_s - \rho_f)\, g/\mu^2$
C_1,C_2 = two constants in Wen and Yu type equation shown in Eq. 6
C_D = drag coefficient
$(C_D)_\epsilon$ = modified drag coefficient defined in Eq. 16
d_p = particle diameter
d_{pi} = particle diameter of size fraction i
g = gravitational acceleration
L = bed height
ΔP = pressure drop across the fluidized bed
Re = Reynolds number, $Re = d_pU\rho_f/\mu$
$(Re)_\epsilon$ = modified Reynolds number defined in Eq. 15
$(Re)_{mf}$ = Reynolds number based on minimum fluidization velocity defined in Eq. 7
U = superficial fluidization velocity
U_{mf} = superficial velocity at minimum fluidization
x_i = weight fraction of size fraction i

Greek Letters

ρ_f = fluid density
ρ_s = particle density
μ = fluid viscosity
ϵ = voidage
ϵ_{mf} = voidage at minimum fluidization
ϕ = shape factor

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